TUM-HEP-428/01

Coupling the Superstring to a D-Brane Ramond-Ramond Background

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Abstract

A new approach to couple the type II superstring to the Ramond-Ramond background of D-branes is proposed. Its construction, using the Ramond-Neveu-Schwarz formalism, is based on the unbroken translations of flat D-branes. It is substantiated by a relation between strings in Ramond-Ramond states and the transversally polarized open strings of the D-branes.

Key words: Ramond-Ramond Background, RNS Formalism, D-Branes
PACS: 11.25.-w

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1 Introduction

D-branes play a special role in the recent development of string theory. They were, for example, important for realizing the duality relations between the five consistent string theories and give rise to phenomenologically interesting models. For some of them, it is desirable to know about string theory with the background fields of D-p-branes present. The major difficulty hereby is the Ramond-Ramond (RR) background. In the standard Ramond-Neveu-Schwarz (RNS) formalism, the usual way to couple the string to this background fails as superconformal invariance is spoiled when the RR vertex operators are used to deform the action.

Despite of this there exist some works, e.g. [1,2], in the RNS formalism. Other approaches [3–5] use either the Green-Schwarz (GS) formalism or a mixture between RNS and GS formalism, like the hybrid formalism. The approach [3] in the GS formalism for the $AdS_5 \otimes S^5$ RR background, for example, is based on considering the coset supermanifold $\mathrm{SU}'(2,2|4)/(\mathrm{SO}(4,1) \otimes \mathrm{SO}(5))$ as target space for the superstring. The coupling to the RR background found this way agrees to some extend with the one constructed by performing double dimensional reduction and a T-duality transformation of the 11-dimensional supermembrane action coupled to the supergravity background [6]. The approach in the hybrid formalism [4] uses the topological string theory in RNS variables and partly introduces GS variables. This way the RR vertex operators can be used to deform the action of the free superstring to yield a coupling to the RR background field.

In this letter I propose a way to couple the superstring to the RR background of a D-brane in the RNS formalism without using the RR vertex operators.

2 Coupling of 1-Forms to the RNS Superstring

For the approach to coupling the superstring to the RR background of a D-brane presented in section 3, it is appropriate to specify the known coupling of 1-forms in the RNS formalism in the presence of D-p-branes.

In the bosonic string theory, the coupling of a 1-form background is just given by integrating the pullback of the background field $A^{(1)}$ to the string boundary

$\partial\Sigma$. With the embedding X and the pullback X^* it can be written as

$$\begin{aligned}\int\limits_{\partial\Sigma} X^* A^{(1)} &= \int\limits_{\partial\Sigma} A^{(1)}(X(z,\overline{z})) = \int\limits_{\partial\Sigma} A^{(1)}_\mu(X(z,\overline{z}))\,\mathrm{d}X^\mu(z,\overline{z}) \\ &= \int\limits_{\partial\Sigma} A^{(1)}_\mu(X(z,\overline{z}))\,\{\partial X^\mu \mathrm{d}z + \overline{\partial}X^\mu \mathrm{d}\overline{z}\}\;. \end{aligned} \tag{1}$$

This changes somewhat if D-p-branes are present. In Cartesian coordinates and with $i\in\{0,\ldots,p\}$ denoting indices parallel and $t\in\{p+1,\ldots,9\}$ indices transverse to the p-brane we get the proper coupling by T-dualizing equation (1):

$$\int\limits_{\partial\Sigma} A^{(1)}_i(X)\,\{\partial X^i \mathrm{d}z + \overline{\partial}X^i \mathrm{d}\overline{z}\} + \int\limits_{\partial\Sigma} \Phi_t(X)\,\{\partial X^t \mathrm{d}z - \overline{\partial}X^t \mathrm{d}\overline{z}\}\;. \tag{2}$$

Φ_t is defined as $\Phi_t := A^{(1)}_t$. Supersymmetrizing equation (2), i.e. substituting the embedding components X^μ by the superfields $\boldsymbol{X}^\mu = X^\mu + \theta\psi^\mu + \overline{\theta}\,\widetilde{\psi}^\mu + \overline{\theta}\theta F^\mu$, the Dolbeaut operators $\partial, \overline{\partial}$ by $\mathrm{D} = \partial_\theta + \theta\partial$, $\overline{\mathrm{D}} = \partial_{\overline{\theta}} + \overline{\theta}\,\overline{\partial}$ and adding the integration over the complex fermionic coordinates $\overline{\theta}, \theta$ yields the coupling

$$\begin{aligned}&\int\limits_{\partial\Sigma} A_\mu(\boldsymbol{X})\,\{\mathrm{D}\boldsymbol{X}^i\,\mathrm{d}z\,\mathrm{d}\theta + \overline{\mathrm{D}}\boldsymbol{X}^i\,\mathrm{d}\overline{z}\,\mathrm{d}\overline{\theta}\,\} \\ &\quad + \int\limits_{\partial\Sigma} \Phi_t(\boldsymbol{X})\,\{\mathrm{D}\boldsymbol{X}^i\,\mathrm{d}z\,\mathrm{d}\theta - \overline{\mathrm{D}}\boldsymbol{X}^i\,\mathrm{d}\overline{z}\,\mathrm{d}\overline{\theta}\,\} \end{aligned} \tag{3}$$

of a 1-form background field to the superstring in the RNS formalism.

3 Geometric Approach to Coupling the RR p-Form to the String

The RR background of a D-p-brane consists of a $p+1$-form potential $C^{(p+1)}$. The unbroken translations of the D-p-brane are generated by the $p+1$ vector fields $V_0 := \frac{\partial}{\partial X^0}, \ldots, V_p := \frac{\partial}{\partial X^p}$.

Generally, from a p-form, one gets a $p+1$-form by taking the exterior derivative and a $p-1$-form by the interior product i_V with a vector field V. Using $i_{V_0}, \ldots, i_{V_p}$ and the exterior derivative d as tools, from $C^{(p+1)}$ an interesting 1-form,

$$i_{V_0}\cdots i_{V_p}\,\mathrm{d}C^{(p+1)} = i_{V_0}\cdots i_{V_p}\,F^{(p+2)}\;, \tag{4}$$

can be constructed. $F^{(p+2)}$ is the $p+2$-form field strength. The pullback of this 1-form can now be geometrically coupled to the string boundary or equivalently the closed 2-form $\mathrm{d}\,i_{V_0}\cdots i_{V_p}\,\mathrm{d}C^{(p+1)}$ can be coupled to the string itself.

Supersymmetrizing and T-dualizing it as in section 2 yields

$$\mathscr{L}_{\text{int}} = \int\limits_{\partial\Sigma} F_{0\dots pt}(\boldsymbol{X}) \, \{ \mathrm{D}\boldsymbol{X}^t \mathrm{d}z \, \mathrm{d}\theta - \overline{\mathrm{D}}\boldsymbol{X}^t \mathrm{d}\overline{z} \, \mathrm{d}\overline{\theta} \} \, . \tag{5}$$

This result is the proposed interaction Lagrangian for the superstring in an external RR background field of a D-p-Brane.

4 Discussion of the Interaction Lagrangian

In general, viable interaction Lagrangians can be constructed using the vertex operators corresponding to the desired background field. This method fails for a RR background. The other way round, the interaction Lagrangian proposed in equation (5), can, in this picture, be identified as a construction from a vertex operator for an open string state polarized vertically to the D-p-brane. This can be seen more explicitly by writing (5) in component fields,

$$\mathscr{L}_{\text{int}} = \int\limits_{\partial\Sigma} \mathrm{d}z \, F_{0\dots pt}(X(z,\overline{z})) \, [\partial X^t(z) + (2 \overleftarrow{\partial_i} \, \psi^i(z))\psi^t(z)] \, , \tag{6}$$

using the notation $F_{0\dots pt}(X) \overleftarrow{\partial_i} := \partial_i F_{0\dots pt}(X)$. Thus, if the proposed coupling of equation (5) is reasonable, transversally polarized open strings should be related to strings in RR states in a special way. Their vertex operators should be a proper substitute for the RR vertex operators.

5 RR States and the Transversally Polarized Open String States

In order to investigate the relation of closed strings in RR states and open strings polarized transversally to a D-brane, I consider the case of a D-p-brane and the string states corresponding to its p+1-form RR background field with a p+2-form field strength. Calculating the disk amplitude [2]

$$\mathscr{A}(V^{\text{RR}}(q)V^{\text{open}}(k))_{\text{D}} = \left\langle c(z)c(\overline{z})V^{\text{RR}}_{(q)}(z,\overline{z}) \, c(w)V^{\text{open}}_{(k)}(w)\right\rangle_{\text{D}} \tag{7}$$

the usual way, using the doubling trick, with the vertex operators

$$V^{\text{RR}}_{(q)} = e^{-\frac{1}{2}\phi} S^{\alpha}(z) \, f_{\mu_1\dots\mu_{p+1}} \Gamma^{\mu_1\dots\mu_{p+1}}_{\alpha\beta} \, e^{-\frac{1}{2}\overline{\phi}} \, \overline{S}^{\beta}(\overline{z}) \, e^{iqX(z,\overline{z})} \tag{8}$$

[2] A recent, detailed calculation of more general disk amplitudes with RR vertex operators in the $(-\frac{1}{2}, -\frac{3}{2})$ ghost picture can be found in [7].

for the RR state in the $(-\frac{1}{2}, -\frac{1}{2})$ ghost picture (S^α are the spin fields) and

$$V^{\rm open}_{(k)} = \xi_t \, e^{-\phi} \psi^t \, e^{i2k_\| X}(w) \tag{9}$$

for the transversally polarized open string state in the (-1) ghost picture, gives

$$\mathscr{A}(V^{\rm RR}(q) V^{\rm open}(k))_{\rm D} \sim f_{0\dots pt} \, \xi^t \, \delta(2q_\| + 2k_\|) \ . \tag{10}$$

Thus, open strings with a transverse polarization can turn into closed strings in RR states with the special polarization $f_{0\dots pt} \neq 0$ and vice versa. Therefore, it is reasonable to investigate the possibility of mediating the effect of a RR background by the open strings.

6 Conclusions

Using the unbroken translations of a flat D-p-Brane, from its p-form RR background a 1-form is constructed and coupled in the usual way to the boundary of the string worldsheet. After supersymmetrizing and T-dualizing, the result in equation (5) is a new way to couple the type II superstring to the RR background of a D-brane. This coupling can be interpreted as mediating the RR background by open strings polarized transversally to the D-brane. It is substantiated by the result that strings in RR states corresponding to the RR background mix with the above mentioned open string states, as given in equation (10).

Thus, the proposed complete action for the type II superstring in the RR background of a D-brane including the usual gravitational $G_{\mu\nu}$ and Dilaton ϕ background is given by

$$\begin{aligned} S = {} & \frac{1}{2} \int\limits_{\Sigma} {\rm d}^2 z \, {\rm d}^2\theta \, G_{\mu\nu}(\boldsymbol{X}) \, \overline{\rm D} \boldsymbol{X}^\mu {\rm D} \boldsymbol{X}^\nu + \int\limits_{\Sigma} {\rm d}^2 z \, \phi(X) R^{(2)} \\ & + \int\limits_{\partial\Sigma} F_{0\dots pt}(\boldsymbol{X}) \, \{ {\rm D}\boldsymbol{X}^t \, {\rm d}z \, {\rm d}\theta - \overline{\rm D}\boldsymbol{X}^t \, {\rm d}\bar{z} \, {\rm d}\bar{\theta} \} \ . \end{aligned} \tag{11}$$

To study a general RR background in this approach, a background field expansion has to be performed at the desired location in target space.

Acknowledgements

I would like to thank I. Sachs, C. Scrucca, E. Scheidegger and S. Theisen for useful discussions. This work was supported by the "Sonderforschungsbereich 375 für Astro-Teilchenphysik der Deutschen Forschungsgemeinschaft".

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